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## ABSTRACT

The mathematical derivation of the statistics used for inference in some linear models assumes that the values of the independent variables are measured without error. This assumption is often disregarded when these models are utilized in research. This study is an investigation of the consequences of the violation of this assumption for one family of linear models on the magnitude of these statistics and the frequency of Type I error in these models. The results of this study indicate that although the frequency of Type I error in these models is relatively unaffected, the values of the statistics are seriously affected by the concomitant variable being other than error-free. (Author)

## ABSTRACT

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AN EMPIRICAL INVESTIGATION OF SOME EFFECTS OF THE VIOLATION OF  
THE ASSUMPTION THAT THE COVARIABLE IN ANALYSIS OF COVARIANCE  
IS MEASURED WITHOUT ERROR

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The mathematical derivation of the statistics used for inference in some linear models assumes that the values of the independent variables are measured without error. This assumption is often disregarded when these models are utilized in research. This study is an investigation of the consequences of the violation of this assumption for one family of linear models on the magnitude of these statistics and the frequency of Type I error in these models. The results of this study indicate that the frequency of Type I error in these models is relatively unaffected, and the values of the statistics are probably not seriously affected by the concomitant variable being other than error free. However, an inadequacy in the procedure cause any conclusions to be of a tentative nature.

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R. A. Fisher, who conceived the statistical technique analysis of covariance (Fisher, 1932), stated that "it [analysis of covariance] combines the advantages and reconciles the requirements of two widely applicable procedures known as regression and analysis of variance" (Fisher, 1973, p. 283). Although this combination of regression and analysis of variance is obvious in most presentations of analysis of covariance from an applications point of view, the contributions of regression to the set of assumptions for analysis of covariance are seldom mentioned. This study involves two of the often neglected assumptions of analysis of covariance which stem from the role played by regression.

In a linear model context analysis of covariance can be thought of as a data reduction procedure in which characteristics or performance of objects or individuals can be described in terms of statistics which occur as coefficients in an appropriate linear model and as an inferential procedure for contrasting certain of the statistics which have been obtained by the solution of the model for a particular set of data. In such a data

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<sup>1</sup>This paper represents a report of one aspect of a continuing empirical research effort concerning certain aspects of the analysis of covariance procedure. The author would like to recognize and thank the following persons for their various contributions to this research: Marshall Adams, Jacqueline W. Calkins, Lorraine Matteson, Eugene Schuster, and the computation center staff at The University of Texas at El Paso.

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reduction procedure, the statistics describing the characteristics or performances of objects or individuals classified in a particular manner by a given value or given values of the independent variable or variables can be seen to be based on the attributes of a joint frequency distribution of the dependent variable and concomitant variable(s). Such statistics are obtainable through the mathematical fit of an appropriate function to the space generated by the data occurring for the dependent variable and concomitant variable(s) for each grouping of objects or individuals defined by a particular value of the independent variable or by particular values of the independent variables.

Comparison of objects or individuals classified in a particular manner by a value of the independent variable or values of the independent variables can be visualized geometrically as comparison of these joint frequency distributions and more specifically as comparison of the attributes of the mathematical functions obtained for each frequency distribution. Such comparisons of these groups can be quantitatively accomplished in terms of assessing the probability that differences among the appropriate statistics which occur in the linear model as summaries of the frequency distributions do not exist for the populations from which the groups were obtained. These probabilities can be obtained through evaluation of  $F$  statistics which are calculable as functions of the amount of error in one model in which differences in the appropriate statistics are allowed to exist and another model which has been constrained to eliminate the differences to be examined.

The simplest case of analysis of covariance is the most usual application of this technique. In this case there is a single concomitant variable and a single independent variable, and a linear relationship is the most appropriate mathematical function with which to fit the data for each group. Since the latter condition implies that the relationship between the dependent variable

and the concomitant variable is linear, it can be readily seen that a true model for this case must contain an intercept and a slope for each value of the independent variable. Obviously, comparisons among the various groups, which are defined by the independent variable, must be accomplished in terms of differences among the group slopes and intercepts.

The following procedure is intended to demonstrate how such comparisons can be accomplished when certain assumptions involving the variables are met. To test for differences among the group slopes, the F statistic from the expression

$$F_{\text{slopes}} = \frac{(q_2 - q_1) / df_1}{q_1 / df_2}$$

can be calculated and evaluated where  $q_1$  is the error sum of squares for the true model which contains a slope and intercept for each group;  $q_2$  is the error sum of squares for a model which contains an intercept estimate for each group but the same slope estimate for all groups (this restricted model constrains each group to have the same slope, thus eliminating all group slope differences);  $df_1$  is a number with a magnitude one less than the number of groups; and  $df_2$  is a number with a magnitude equal to the summation of the number in each group reduced by two (Ward and Jennings, 1973, pp. 138-148). To test for differences among the group intercepts when the group slopes are not different, the F statistic from the expression

$$F_{\text{intercepts}} = \frac{(q_3 - q_2) / df_1}{q_2 / df_3}$$

can be calculated and evaluated where  $q_2$  is the error sum of squares for the restricted model presented above;  $q_3$  is the error sum of squares for a model which contains the same intercept estimate and the same slope estimate for each group (this further restricted model constrains each group to have the same intercept and the same slope, thus eliminating all group intercept and

group slope differences);  $df_1$  is a number (as above) with a magnitude one less than the number of groups; and  $df_3$  is a number with a magnitude equal to the summation of the number in each group reduced by a number one less than the number of groups (Ward and Jennings, 1972, pp. 138-148).

Although particular interpretations of differences among group slopes and intercepts depend upon particular situations, some general interpretations are as follows:

1. The occurrence of differences in the group slopes implies that the rate of change of the dependent variable with respect to the concomitant variable differs for the various groups.
2. The occurrence of differences in the group intercepts when the group slopes are not different implies that there are constant differences among the groups in terms of the dependent variable for each value of the concomitant variable.

Generally comparisons of group intercepts are of little interest if the group slopes are different, because in this situation differences among the groups are variable, since the difference depends on particular values of the concomitant variable.

In order to show the inherent distributional assumptions involving the variables for analysis of covariance when there is only a single concomitant variable, the relationship between the dependent and concomitant variables is linear, the group slopes are not different, and there is only one independent variable, the following model for analysis of covariance as presented by Graybill (1961, pp. 383-384) will be used. "The model is

$$y_{ij} = \mu + \tau_i + \beta x_{ij} + e_{ij} \quad \begin{matrix} j = 1, 2, \dots, r \\ i = 1, 2, \dots, t \end{matrix}$$

where  $y_{ij}$  is an observed random variable;  $x_{ij}$  is an observed fixed quantity, usually called a concomitant variable;  $\mu$  is a constant;  $\tau_i$  is the  $i$ th treatment constant;  $\beta$  is the regression coefficient (unknown); and  $e_{ij}$  are unobserved

normal variables that are independent and have means 0 and constant variances  $\sigma^2$ ." Examination of this model and its assumed distributional properties reveals that (1) the dependent variable  $y$  is assumed to be a random variable measured without error; (2) the concomitant variable  $x$  is assumed to be a fixed variable measured without error; and (3)  $\beta$  is a constant for all treatment groups. The model is essentially the one discussed previously in which all groups were constrained to have the same group slope.

In addition, Graybill (1961, pp. 394-396) shows that when the data conform to the distributional assumptions of the model, many desirable mathematical properties result. It is only as a consequence of these resulting mathematical properties that the descriptive and inferential uses of analysis of covariance are viable. Unfortunately, it seems that little pertinent research by the mathematical statisticians is available to indicate what consequences are to be expected when the data do not conform to the distributional assumptions of the model.

For the usual application of such models by researchers, the assumptions involving error free measurement of the dependent and concomitant variables and the necessity that the concomitant variable be a fixed variable are often neglected. Previous research (Calkins and Jennings, 1972) has indicated that the consequences of the violation of the assumption that the concomitant variable is fixed are probably of little practical importance to investigators concerned with estimation of slopes and intercepts and Type I error resulting from contrasts of slopes and intercepts. In the research presented in this paper, some of the consequences of the violation of the assumptions that the concomitant variable is quantified in an error free manner and that the concomitant variable is fixed and quantified in an errorless manner have been investigated.

# PROCEDURE

This research was conducted in much the same manner using essentially the same computer programs as the previous investigation in this series (Calkins and Jennings, 1972). Since complete details of the procedure were presented in the previous investigation, this procedure section will be abbreviated, and persons interested in more details may consult that paper.

The set of models investigated are those discussed previously in this paper in which the true model contains one independent variable and one concomitant variable which is linearly related to the dependent variable. The investigation of the effects which occur in these models when the concomitant variable contains error, or contains error and is not a fixed variable, was restricted to possible disturbance of (1) the distributions of the slopes and intercepts, and (2) the distributions of the  $F$  statistic utilized for decisions concerning possible differences in the slopes and intercepts. Possible disturbance of the distributions of the slopes and intercepts were examined through the calculation of functions of the first four cumulative moments of their respective distributions, the mean, variance, skewness and kurtosis. Theoretically it is expected that the slopes and intercepts will be distributed normally with the following values:

$$\text{Expected mean for slopes} = \frac{\rho_{xy} \cdot \sigma_y}{\sigma_x}$$

$$\text{Expected variance for slopes} = \frac{\sigma_y^2 (1 - \rho_{xy}^2)}{\sigma_x^2 (n - 2)}$$

$$\text{Expected mean for intercepts} = \mu_x - \frac{\rho_{xy} \sigma_y}{\sigma_x} \mu_y$$

$$\text{Expected variance for intercepts} = \frac{n}{2(n-1)^2} \frac{\sigma_y^2}{\sigma_x^2} (1 - \rho_{xy}^2) (\sigma_x^2 + \mu_x^2)$$

(approximate formula)



where  $n$  is the sample size

$\rho_{xy}$  is the correlation for the population

$\mu_x$  and  $\mu_y$  are the marginal population means for the  $x$  and  $y$  populations

$\sigma_x$  and  $\sigma_y$  are the marginal population standard deviations for the  $x$  and  $y$  populations.

Possible disturbance of the distributions of the  $F$  statistic utilized for decisions concerning possible differences in the slopes and intercepts were examined through comparison of the mean of the observed sampling distributions with the theoretical values and comparison of the number of observed values for the distributions of  $F$ 's for slope and intercept which were equal to or greater than the specific values of the central  $F$  distribution for the proper degrees of freedom at the .05 and .01 confidence levels. Since in this investigation all the samples were drawn from the same population, the expected values of  $F_{\text{intercept}}$  should be near  $df_3/(df_3 - 2)$  and  $F_{\text{slope}}$  should be near  $df_2/(df_2 - 2)$  where  $df_2$  and  $df_3$  were previously defined in this paper. In addition, approximately five per cent of the values observed for  $F_{\text{intercept}}$  and  $F_{\text{slope}}$  should be equal to or greater than the specific values of the central  $F$  distribution for the appropriate degrees of freedom at the .05 level and approximately one per cent should be equal to or greater than the equivalent value at the .01 level.

This empirical research, which was performed with the indispensable help of a digital computer, is of the Monte Carlo type. The computer system utilized was a CDC 3150 located at The University of Texas at El Paso. The FORTRAN programs written for this series of investigations function to (1) generate multiple pairs of values which simulate random samples of  $X, Y$  pairs from bivariate populations of values with various characteristics; (2) combine these pairs of values into samples of specified size; (3) calculate various quantitative indices on the data for and between the samples which correspond

to the quantitative indices resulting from the descriptive and inferential use of the linear models presented previously; (4) tabulate these indices for a specified number of experiments, each including a specified number of samples, in order to obtain a frequency distribution for each index; (5) calculate descriptive characteristics for each frequency distribution; and (6) produce output showing the frequency distributions and their descriptive characteristics. In this procedure, error in the covariable was introduced by generating a pair of X and Y values which had suitable characteristics for a certain condition of the experiment and then adding a component of error which was a random normal deviate from a population with zero mean and specified variance to each X value.

The characteristics of the bivariate populations which are specifiable with this set of programs and which were varied in this investigation are (1) sampling procedure for the covariable (random or fixed); (2) distribution shape of the covariable (normal or rectangular); (3) amount of error in the covariable (in terms of the standard error of measurement as a function of percent of standard deviation of Y -- 0%, 33%, 67%, and 100%); (4) variance of the Y values in each X array (in terms of standard error of estimate as a function of correlations of .0, .30, .60, and .90); and (5) sample size (samples of 5, 13 and 39 X,Y pairs were used). The characteristics of the bivariate populations which are specifiable but were not varied in this investigation are (1) means of the X and Y marginal distributions (maintained at or near 50); (2) number of groups obtained for each experiment (each experiment consisted of the data obtained for two groups with each group containing either 5, 13 or 39 X,Y pairs); and (3) number of experiments (1000 experiments were conducted for each of the possible combinations of the characteristics of the bivariate populations).

It was intended that the standard deviations of the X and Y marginal distributions be maintained at or near 10., but procedural constraints interfered to some extent. However, as an attempt to compensate for this lack of control, two sets of theoretically expected values for the mean and variance of the distributions of slope and intercept are presented in the Results section; one set is based on the intended population values and the other set is based on observed values. Comparison of these values should indicate to some degree the extent of this problem or the adequacy of the entire procedure.

Since it is always desirable in any scientific enterprise to maintain constant as many variables as possible, in this series of investigations it would be desirable to maintain constant the X variance for all experimental conditions. However, these constant variances are difficult to achieve, especially for the fixed X condition (at least from the author's current cognitive set). This difficulty occurs because for the fixed X case it must be decided whether it is more important to maintain a constant variance for the covariable or to use the same fixed X values for the various sample sizes. In the first investigation in this series (Calkins and Jennings, 1972), the same set of X values were used for all the fixed X cases for the two distribution shapes by varying the frequency of occurrence of each X value, but in this procedure the variances of the X values were allowed to vary. (The peculiar choice of X,Y pairs per sample -- 5, 13 and 39 -- is a byproduct of this procedure.) In the present investigation, a compromise between maintaining constant variance and the same set of fixed X values was effected, and as a consequence neither objective was fully attained. In this investigation fairly constant variance for the zero error condition was achieved by allowing the set of fixed X's to differ for different sample sizes, but when the X values contained a component of error the variance of the X values was to some extent uncontrolled. Some control over the variance

in this procedure could be established by a linear transformation of the  $X$  values containing error by subtracting a different constant for each quantity of error and each sample size, but such a transformation would destroy the concept of one set of fixed  $X$ 's for all experimental conditions. (The author would appreciate comments concerning this problem.)

## RESULTS

The results are presented in Tables 2 through 25. Tables 2-5 present the results of the experiments involving the investigation of the distributions of slopes for the several experimental conditions for the concomitant variable normally distributed with each table representing a particular value of error of estimate. Tables 6-9 contain the same information as Tables 2-5 except that in these tables the concomitant variable is uniformly (rectangularly) distributed. Tables 10-13 present the results of experiments involving the investigation of the distributions of intercept for the several experimental conditions for the concomitant variable normally distributed with each table representing a particular value of error of estimate. Tables 14-17 contain the same information as Tables 10-13 except that in these tables the concomitant variable is uniformly (rectangularly) distributed. Tables 18-21 present the results involving the critical statistics for comparison of slopes and intercepts for the several experimental conditions for the concomitant variable normally distributed with each table representing a particular value of error of estimate. Tables 22-25 contain the same information as Tables 18-21 except that in these tables the concomitant variable is uniformly (rectangularly) distributed. Table 1 presents explication of the terms and abbreviations found in Tables 2-25.

Proper interpretation of these results can reveal the consequences of three aspects of the violation of assumptions concerning the concomitant

TABLE 1

Explication of Terms and Abbreviations  
Found in Tables 2 through 25

1. Expect T - expected values based on specified characteristics of the bivariate populations
2. Expect O - expected values based on observed characteristics of the populations (example - the expected value for intercepts is calculable by subtracting the mean of the frequency distribution containing 2000 slopes multiplied by the mean of the frequency distribution containing 2000 X marginal means from the mean of the frequency distribution containing 2000 Y marginal means)
3. Error - standard error of measurement as a function of percent of standard deviation of Y values
4. RN - values of X (the concomitant variable) were obtained by random sampling from a normal distribution
5. FN - values of X (the concomitant variable) were preselected (fixed) such that they have zero skewness and kurtosis
6. RR - values of X (the concomitant variable) were obtained by random sampling from a uniform (rectangular) distribution
7. FR - values of X (the concomitant variable) were preselected (fixed) such that the values have equal frequency (rectangularly distributed)
8. Mean F - mean value of the observed frequency distribution of F values
9. I-Error - number of Type I errors observed at the .05/.01 level of significance

2. Since the data are normally distributed, the expected values of the sample means, variances, skewness, and kurtosis are 0.0, 0.0000, 0.0000, and 0.0000, respectively. The expected values of the sample means, variances, skewness, and kurtosis are 0.0, 0.0000, 0.0000, and 0.0000, respectively.

### SAMPLE SIZE

N=5		N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000	0.0000	0.0000

N=5

N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000

N=13

N=39		N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000	0.0000	0.0000

N=39

N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000

N=13

N=39		N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000	0.0000	0.0000

N=39

N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000

N=13

N=39		N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000	0.0000	0.0000

N=39

N=13		N=39	
MEAN	EXPECT T	EXPECT T	EXPECT T
0.0	0.0000	0.0	0.0000
VAR	0.0000	0.0000	0.0000
SKEN	0.0000	0.0000	0.0000
KURT	0.0000	0.0000	0.0000

N=13

52,001 5175

[illegible]

9. 500 INHABITANTS IN THE SAMPLE DISTRICTS. 10. 500 FOR SEPARATE POPULATIONS, 1000 FOR VALLEY OF C. 11. 5000 FOR VALLEYS, 10000 FOR COUNTRY, AND A VALUE OF 100000 FOR THE ENTIRE POPULATION OF 1900.

$$V = 33$$
[illegible]



FILED

[illegible]

—

[illegible]

33  
33

MEAN	0.9000	0.8756	0.8318	0.9000	0.7693	0.7834	0.9000	0.6762	0.6894
VAR	0.2517	0.2551	0.2522	0.1314	0.1644	0.1711	0.0717	0.0912	0.1194
SEMEAN	0.0	0.0	0.5013	0.0	0.0	0.4654	0.0	0.0	0.2293
SEVAR	0.0	0.0	0.6313	0.0	0.0	0.6940	0.0	0.0	0.1388
MEAN	0.9000	0.5312	0.5243	0.9000	0.6768	0.6130	0.9000	0.6146	0.6184
VAR	0.2517	0.3435	0.5320	0.1314	0.1712	0.1706	0.0717	0.0915	0.0894
SEMEAN	0.0	0.0	0.2910	0.0	0.0	0.0351	0.0	0.0	-0.0328
SEVAR	0.0	0.0	3.6232	0.0	0.0	0.5498	0.0	0.0	0.0040

$$19 = 67$$

	MEAN	0.9000	0.5216	0.6788	0.9000	0.5121	0.5294	0.9000	0.3752	0.3379
	WAS	0.2517	0.3035	0.3337	0.1314	0.1751	0.1747	0.0717	0.0375	0.1233
	WSES	0.00	0.00	0.3725	0.00	0.00	0.4596	0.00	0.00	0.0938
	WJPT	0.00	0.00	7.2520	0.00	0.00	1.0038	0.00	0.00	0.0721

  

	MEAN	0.9000	0.4107	0.4674	0.9000	0.4498	0.9000	0.4454	0.4491
	WAS	0.2517	0.3353	0.3393	0.1314	0.1601	0.1734	0.0717	0.0903
	WSES	0.00	0.00	-0.2523	0.00	0.00	0.0539	0.00	0.00
	WJPT	0.00	0.00	3.6377	0.00	0.00	0.9697	0.00	0.00

0.1000

$$C = I$$

5. SOME CURRENT ISSUES IN THE SAMPLING DISTRIBUTION OF SLOPE FOR  $X$  VALUES RECTANGULARLY DISTRIBUTED ON  $X$  VALUES NON-UNIFORMLY DISTRIBUTED ON  $X$ , RANDOM AND FIXED VALUES OF  $Y$  AND SOME COMMENTS ON THE DISTRIBUTION OF THE ESTIMATE BASED ON A CORRELATION COEFFICIENT.

3715 - 10005

N=5			N=13			N=39		
	EXPECT T	EXPECT D	EXPECT T	EXPECT D	EXPECT T	EXPECT D	EXPECT T	EXPECT D
MEAN	0.0	-0.0010	0.0	0.0002	0.0	0.0011	0.0	-0.0032
VAR	0.5774	0.5502	0.3015	0.2979	0.3120	0.3144	0.1642	0.1640
SEME	0.0	0.0	0.0	0.0	0.0381	0.0	0.0	-0.0461
JUST	0.0	0.0	0.0	0.0	0.4799	0.0	0.0	0.9765

—

Variable	Mean	SD	Min	Max	Skewness	Kurtosis	Shapiro-Wilk	Normality
Age	35.2	12.5	18	65	0.15	3.2	0.98	0.95
Gender	1.2	0.4	1	2	-0.05	3.5	0.99	0.98
Education	12.5	2.1	9	16	-0.10	3.8	0.97	0.93
Income	2500	1500	500	6000	0.20	4.5	0.95	0.90
Marital Status	1.5	0.5	1	3	-0.02	3.1	0.99	0.97
Occupation	2.8	1.2	1	5	0.10	3.3	0.96	0.92
Health Status	1.8	0.6	1	3	-0.08	3.4	0.98	0.96
Stress Level	3.2	1.5	1	5	0.18	4.2	0.94	0.88
Life Satisfaction	4.5	1.2	3	6	-0.05	3.6	0.97	0.94
Resilience Score	2.1	0.9	1	4	0.12	3.7	0.96	0.91
Emotional Stability	3.8	1.1	2	5	-0.03	3.9	0.98	0.95
Overall Well-being	3.5	1.0	2	5	0.08	4.0	0.95	0.90

	0.0	0.0133	0.0139	0.0	-0.0023	-0.0015	0.0	0.0014	0.0005
MEAN	0.0	0.0133	0.0139	0.0	-0.0023	-0.0015	0.0	0.0014	0.0005
VAR	0.5774	0.5376	0.6455	0.3015	0.2840	0.2549	0.1644	0.1556	0.1611
SKEW	0.0	0.0	-0.0665	0.0	0.0	-0.0410	0.0	0.0	-0.0656
KURT	0.0	0.0	4.7423	0.0	0.0	3.2406	0.0	0.0	-0.1962

23

[illegible]

	mean	s.d.	var	skew	kurt	neg loglik	aic	bic	log post
0.0	-0.0045	0.0	0.0053	0.0	0.010	-0.0008	0.0	0.0012	0.0010
0.5774	0.4778	0.5492	0.3015	0.2503	0.2616	0.2616	0.1644	0.1363	0.1396
0.0	0.0	0.0545	0.0	0.0	-0.0435	0.0	0.0	0.0	0.0479
0.0	0.0	0.6314	0.0	0.0	0.5606	0.0	0.0	0.0	0.1637

17-11-17

[illegible]

	Mean	Var	Skew	Kurt	Shapiro-Wilk	Anderson-Darling	Normality
1	0.0068	0.0151	0.0	0.0	0.9972	0.0078	0.0023
2	0.4971	0.4537	0.0	0.3015	0.2130	0.2233	0.1159
3	0.0	0.1121	0.0	0.0	0.0	0.3746	0.0
4	0.0	7.0302	0.0	0.0	0.0	0.7579	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37	0.0	0.0	0.0	0.0	0.0	0.0	0.0
38	0.0	0.0	0.0	0.0	0.0	0.0	0.0
39	0.0	0.0	0.0	0.0	0.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	0.0	0.0
41	0.0	0.0	0.0	0.0	0.0	0.0	0.0
42	0.0	0.0	0.0	0.0	0.0	0.0	0.0
43	0.0	0.0	0.0	0.0	0.0	0.0	0.0
44	0.0	0.0	0.0	0.0	0.0	0.0	0.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0
46	0.0	0.0	0.0	0.0	0.0	0.0	0.0
47	0.0	0.0	0.0	0.0	0.0	0.0	0.0
48	0.0	0.0	0.0	0.0	0.0	0.0	0.0
49	0.0	0.0	0.0	0.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
51	0.0	0.0	0.0	0.0	0.0	0.0	0.0

1

[illegible]

TABLE 7 SOME CHARACTERISTICS OF THE SAMPLING DISTRIBUTION OF SUMS OF X VALUES RECTANGULARLY DISTRIBUTED BUT WITH THE Y VALUES NORMALLY DISTRIBUTED FOR EVERY VALUE OF X, -RANGE AND FIXED VALUES OF THE SAMPLE SIZES, FOR RATIOS OF ERROR, AND A VALUE OF ERROR OF ESTIMATE BASED ON A CORRELATION OF 0.30

## SAMPLE SIZE

N=5				N=13				N=39			
	EXPECT T	EXPECT O	OBSERVED	EXPECT T	EXPECT O	OBSERVED	EXPECT T	EXPECT O	OBSERVED		
MEAN	0.3000	0.2679	0.2995	0.3000	0.3024	0.3137	0.3000	0.3000	0.2978		
VAR	0.5503	0.5460	0.6007	0.2376	0.2821	0.2853	0.1568	0.1568	0.1565		
SKFW	0.0	0.0	0.1613	0.0	0.0	-0.1122	0.0	0.0	-0.0125		
KURT	0.0	0.0	4.3944	0.0	0.0	0.5110	0.0	0.0	0.1141		
ERROR= 0											
MEAN	0.3000	0.2742	0.3063	0.3000	0.2939	0.3040	0.3000	0.3010	0.3026		
VAR	0.5503	0.5151	0.4753	0.2876	0.2733	0.2639	0.1568	0.1568	0.1523		
SKFW	0.0	0.0	0.0520	0.0	0.0	0.1444	0.0	0.0	0.0222		
KURT	0.0	0.0	-0.0315	0.0	0.0	0.0490	0.0	0.0	-0.1556		
ERROR= 33											
MEAN	0.3000	0.2216	0.2514	0.3000	0.2537	0.2697	0.3000	0.2571	0.2713		
VAR	0.5503	0.5063	0.6054	0.2876	0.2729	0.2751	0.1568	0.1501	0.1505		
SKFW	0.0	0.0	-0.2467	0.0	0.0	0.0386	0.0	0.0	0.0913		
KURT	0.0	0.0	10.3975	0.0	0.0	0.3247	0.0	0.0	0.0885		
ERROR= 67											
MEAN	0.3000	0.2461	0.2958	0.3000	0.2614	0.2737	0.3000	0.2551	0.2599		
VAR	0.5503	0.5020	0.4768	0.2876	0.2613	0.2641	0.1568	0.1544	0.1507		
SKFW	0.0	0.0	0.2139	0.0	0.0	0.0893	0.0	0.0	0.0347		
KURT	0.0	0.0	0.4467	0.0	0.0	0.2244	0.0	0.0	0.1770		
ERROR= 100											
MEAN	0.3000	0.1752	0.2059	0.3000	0.2038	0.2145	0.3000	0.2070	0.2107		
VAR	0.5503	0.4552	0.5290	0.2876	0.2409	0.2519	0.1568	0.1316	0.1350		
SKFW	0.0	0.0	0.2369	0.0	0.0	0.1337	0.0	0.0	-0.0186		
KURT	0.0	0.0	3.4234	0.0	0.0	0.7654	0.0	0.0	0.1101		
ERROR= 133											
MEAN	0.3000	0.2075	0.2469	0.3000	0.2125	0.2253	0.3000	0.1638	0.1683		
VAR	0.5503	0.4492	0.4574	0.2876	0.2341	0.2417	0.1568	0.1270	0.1300		
SKFW	0.0	0.0	0.2227	0.0	0.0	0.1501	0.0	0.0	0.1050		
KURT	0.0	0.0	0.8334	0.0	0.0	0.2096	0.0	0.0	0.3206		
ERROR= 167											
MEAN	0.3000	0.1603	0.1694	0.3000	0.1497	0.1560	0.3000	0.1454	0.1472		
VAR	0.5503	0.3980	0.4774	0.2876	0.2105	0.2146	0.1568	0.1137	0.1121		
SKFW	0.0	0.0	0.4942	0.0	0.0	0.0030	0.0	0.0	0.0271		
KURT	0.0	0.0	5.6341	0.0	0.0	0.4656	0.0	0.0	-0.0637		
ERROR= 200											
MEAN	0.3000	0.1537	0.1581	0.3000	0.1442	0.1523	0.3000	0.1021	0.1043		
VAR	0.5503	0.4012	0.4025	0.2876	0.2001	0.2036	0.1568	0.0956	0.0999		
SKFW	0.0	0.0	0.6694	0.0	0.0	0.1010	0.0	0.0	0.1068		
KURT	0.0	0.0	9.1310	0.0	0.0	0.5023	0.0	0.0	0.5524		

TABLE 3. SOME SUMMARY STATISTICS OF THE SAMPLING DISTRIBUTION OF SLOPE FOR X VALUES REGULARLY SPACED AT 0.5 UNITS, AND THE Y VALUES ARE ONLY OBSERVED FOR EVERY VALUE OF X, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0. ESTIMATE BASED ON A CORRELATION OF 0.60

SAMPLE SIZE

	N=5				N=13				N=39			
	MEAN	VAR	SKEN	KURT	MEAN	VAR	SKEN	KURT	MEAN	VAR	SKEN	KURT
N=0	0.5000	0.5406	0.5895	0.5796	0.5000	0.5406	0.5895	0.5796	0.5000	0.5406	0.5895	0.5796
	0.4619	0.4714	0.5125	0.2412	0.4619	0.4714	0.5125	0.2412	0.4619	0.4714	0.5125	0.2412
	0.00	0.00	-0.5311	0.00	0.00	0.00	-0.5311	0.00	0.00	0.00	-0.5311	0.00
	0.00	0.00	11.1175	0.00	0.00	0.00	11.1175	0.00	0.00	0.00	11.1175	0.00
N=33	0.5000	0.5515	0.5925	0.5812	0.5000	0.5515	0.5925	0.5812	0.5000	0.5515	0.5925	0.5812
	0.4619	0.4448	0.4111	0.2412	0.4619	0.4448	0.4111	0.2412	0.4619	0.4448	0.4111	0.2412
	0.00	0.00	0.0640	0.00	0.00	0.00	0.0640	0.00	0.00	0.00	0.0640	0.00
	0.00	0.00	0.0043	0.00	0.00	0.00	0.0043	0.00	0.00	0.00	0.0043	0.00
N=67	0.5000	0.4969	0.5397	0.5372	0.5000	0.4969	0.5397	0.5372	0.5000	0.4969	0.5397	0.5372
	0.4619	0.4571	0.5385	0.2412	0.4619	0.4571	0.5385	0.2412	0.4619	0.4571	0.5385	0.2412
	0.00	0.00	-3.3034	0.00	0.00	0.00	-3.3034	0.00	0.00	0.00	-3.3034	0.00
	0.00	0.00	66.0961	0.00	0.00	0.00	66.0961	0.00	0.00	0.00	66.0961	0.00
N=100	0.5000	0.5239	0.5784	0.5380	0.5000	0.5239	0.5784	0.5380	0.5000	0.5239	0.5784	0.5380
	0.4619	0.4238	0.4112	0.2412	0.4619	0.4238	0.4112	0.2412	0.4619	0.4238	0.4112	0.2412
	0.00	0.00	0.1275	0.00	0.00	0.00	0.1275	0.00	0.00	0.00	0.1275	0.00
	0.00	0.00	0.2329	0.00	0.00	0.00	0.2329	0.00	0.00	0.00	0.2329	0.00
N=133	0.5000	0.3911	0.4363	0.4053	0.5000	0.3911	0.4363	0.4053	0.5000	0.3911	0.4363	0.4053
	0.4619	0.3114	0.4585	0.2171	0.4619	0.3114	0.4585	0.2171	0.4619	0.3114	0.4585	0.2171
	0.00	0.00	0.5362	0.00	0.00	0.00	0.5362	0.00	0.00	0.00	0.5362	0.00
	0.00	0.00	5.0615	0.00	0.00	0.00	5.0615	0.00	0.00	0.00	5.0615	0.00
N=167	0.5000	0.4065	0.4602	0.4294	0.5000	0.4065	0.4602	0.4294	0.5000	0.4065	0.4602	0.4294
	0.4619	0.4072	0.4222	0.2412	0.4619	0.4072	0.4222	0.2412	0.4619	0.4072	0.4222	0.2412
	0.00	0.00	0.6914	0.00	0.00	0.00	0.6914	0.00	0.00	0.00	0.6914	0.00
	0.00	0.00	2.4312	0.00	0.00	0.00	2.4312	0.00	0.00	0.00	2.4312	0.00
N=200	0.5000	0.2948	0.3295	0.2975	0.5000	0.2948	0.3295	0.2975	0.5000	0.2948	0.3295	0.2975
	0.4619	0.3723	0.4701	0.2412	0.4619	0.3723	0.4701	0.2412	0.4619	0.3723	0.4701	0.2412
	0.00	0.00	-3.7075	0.00	0.00	0.00	-3.7075	0.00	0.00	0.00	-3.7075	0.00
	0.00	0.00	51.2427	0.00	0.00	0.00	51.2427	0.00	0.00	0.00	51.2427	0.00
N=233	0.5000	0.2734	0.3345	0.3076	0.5000	0.2734	0.3345	0.3076	0.5000	0.2734	0.3345	0.3076
	0.4619	0.3694	0.4580	0.2412	0.4619	0.3694	0.4580	0.2412	0.4619	0.3694	0.4580	0.2412
	0.00	0.00	0.5334	0.00	0.00	0.00	0.5334	0.00	0.00	0.00	0.5334	0.00
	0.00	0.00	5.5214	0.00	0.00	0.00	5.5214	0.00	0.00	0.00	5.5214	0.00

## SAMPLE SIZE

	N=5				N=13				N=39			
	MEAN	VAR	SKEW	KURT	MEAN	VAR	SKEW	KURT	MEAN	VAR	SKEW	KURT
N=5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
N=13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
N=39	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
N=100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

## SAMPLE SIZE

N=5

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.4421	49.5050	50.0000	48.3304	48.9327	50.0000	50.0618
VAR	20.1556	20.1261	21.9487	10.8333	10.3300	11.3950	5.9254	5.9303
SKW	0.0	0.0	-0.1732	0.0	0.0	0.0570	0.0	0.0
KURT	0.0	0.0	1.5990	0.0	0.0	0.0984	0.0	0.0

ERROR= 0

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	49.1932	49.1635	50.0000	50.2339	50.2492	50.0000	50.0511
VAR	20.1556	18.9015	17.0099	10.8333	11.9309	11.3150	5.9254	6.0779
SKW	0.0	0.0	0.0951	0.0	0.0	-0.0122	0.0	0.0
KURT	0.0	0.0	-0.0651	0.0	0.0	0.2366	0.0	0.0

ERROR= 0

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.6097	50.1752	50.0000	49.0039	49.0365	50.0000	50.0987
VAR	20.1556	19.0422	19.9734	10.8333	10.3103	11.0422	5.9254	5.6219
SKW	0.0	0.0	-0.2951	0.0	0.0	0.1118	0.0	0.0
KURT	0.0	0.0	1.7296	0.0	0.0	0.3197	0.0	0.0

ERROR= 33

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	49.4593	49.5221	50.0000	50.4611	50.5580	50.0000	49.9445
VAR	20.1556	18.2755	17.1612	10.8333	10.9407	10.6450	5.9254	5.3554
SKW	0.0	0.0	-0.0513	0.0	0.0	0.2051	0.0	0.0
KURT	0.0	0.0	0.6300	0.0	0.0	0.3556	0.0	0.0

ERROR= 33

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.9335	50.9179	50.0000	49.5761	49.5636	50.0000	50.1304
VAR	20.1556	14.8057	17.6379	10.8333	9.1170	9.3149	5.9254	4.9397
SKW	0.0	0.0	-0.1336	0.0	0.0	-0.0755	0.0	0.0
KURT	0.0	0.0	1.3360	0.0	0.0	0.3904	0.0	0.0

ERROR= 67

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	48.9814	49.3670	50.0000	49.7255	49.8536	50.0000	50.2199
VAR	20.1556	15.7496	15.9407	10.8333	9.1253	9.3351	5.9254	4.0666
SKW	0.0	0.0	0.1752	0.0	0.0	0.0372	0.0	0.0
KURT	0.0	0.0	0.2796	0.0	0.0	0.2615	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.8533	50.8973	50.0000	49.6951	49.9107	50.0000	49.9149
VAR	20.1556	14.3198	16.1592	10.8333	7.7554	8.2863	5.9254	4.2614
SKW	0.0	0.0	0.2947	0.0	0.0	-0.0107	0.0	0.0
KURT	0.0	0.0	1.5536	0.0	0.0	0.0497	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.1312	50.3463	50.0000	49.9073	49.8437	50.0000	49.9309
VAR	20.1556	13.6611	15.5547	10.8333	7.4078	8.1050	5.9254	3.1750
SKW	0.0	0.0	0.0427	0.0	0.0	0.0293	0.0	0.0
KURT	0.0	0.0	0.5005	0.0	0.0	0.3100	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED	EXPECT T	OBSERVED
MEAN	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
VAR	20.1556	20.1556	20.1556	10.8333	10.8333	10.8333	5.9254	5.9254
SKW	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
KURT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ERROR= 100

N=13

FIG. 11. SOME CHARACTERISTICS OF THE SAMPLING DISTRIBUTION OF INTERCEPTS FOR BIVARIATE JOINT DISTRIBUTIONS, WITH ASSUMED VALUES OF  $\rho$ , THREE SAMPLE SIZES, FOUR AMOUNTS OF  $\rho$ - $\rho$ , AND A VALUE OF  $\rho$  BASED ON ESTIMATES BASED ON A CORRELATION OF 0.30

## SAMPLE SIZE

	N=5				N=13				N=39				N=121
	EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED	
MEAN	35.0000	34.3461	33.7203	35.0000	34.5459	34.4748	35.0000	34.8728	35.0000	34.8728	35.0000	34.9663	
VAR	19.2273	19.4547	20.3450	10.3343	10.4131	10.5751	5.6525	5.6758	5.6525	5.6758	5.6525	5.8795	
SKW	0.0	0.0	-0.0227	0.0	0.0	-0.1135	0.0	0.0	0.0	0.0	0.0	0.0754	
KURT	0.0	0.0	1.2738	0.0	0.0	0.2979	0.0	0.0	0.0	0.0	0.0	-0.0541	
MEAN	35.0000	35.4477	35.5707	35.0000	35.3324	35.2576	35.0000	35.1564	35.0000	35.1564	35.0000	35.1352	
VAR	19.2273	16.0299	16.5233	10.3343	11.2150	10.9759	5.6525	5.8385	5.6525	5.8385	5.6525	5.6478	
SKW	0.0	0.0	-0.0405	0.0	0.0	0.0192	0.0	0.0	0.0	0.0	0.0	0.0132	
KURT	0.0	0.0	-0.1218	0.0	0.0	0.1704	0.0	0.0	0.0	0.0	0.0	-0.0265	
MEAN	35.0000	36.2631	36.8658	35.0000	36.4326	36.6256	35.0000	36.4364	35.0000	36.4364	35.0000	36.5421	
VAR	19.2273	18.0408	18.4969	10.3343	9.9774	9.8276	5.6525	5.4031	5.6525	5.4031	5.6525	5.6687	
SKW	0.0	0.0	0.1479	0.0	0.0	0.1009	0.0	0.0	0.0	0.0	0.0	-0.1328	
KURT	0.0	0.0	0.9250	0.0	0.0	0.2977	0.0	0.0	0.0	0.0	0.0	-0.0020	
MEAN	35.0000	36.3917	36.8996	35.0000	36.7535	36.8648	35.0000	38.6374	35.0000	38.6374	35.0000	38.9555	
VAR	19.2273	17.2573	16.3065	10.3343	10.6210	10.7272	5.6525	5.1672	5.6525	5.1672	5.6525	5.6700	
SKW	0.0	0.0	0.3599	0.0	0.0	-0.0203	0.0	0.0	0.0	0.0	0.0	-0.0151	
KURT	0.0	0.0	-0.0874	0.0	0.0	0.0457	0.0	0.0	0.0	0.0	0.0	0.0436	
MEAN	35.0000	39.4565	39.9283	35.0000	39.6128	39.6286	35.0000	39.4946	35.0000	39.4946	35.0000	39.4931	
VAR	19.2273	16.6960	17.1802	10.3343	8.3824	9.1452	5.6525	4.8081	5.6525	4.8081	5.6525	4.9300	
SKW	0.0	0.0	0.0142	0.0	0.0	-0.0440	0.0	0.0	0.0	0.0	0.0	-0.1098	
KURT	0.0	0.0	1.5557	0.0	0.0	0.1795	0.0	0.0	0.0	0.0	0.0	-0.1008	
MEAN	35.0000	39.8104	39.4283	35.0000	40.6465	41.0824	35.0000	43.4441	35.0000	43.4441	35.0000	43.6814	
VAR	19.2273	15.5644	15.9439	10.3343	8.3146	9.1660	5.6525	3.9782	5.6525	3.9782	5.6525	5.7022	
SKW	0.0	0.0	-0.4380	0.0	0.0	-0.0093	0.0	0.0	0.0	0.0	0.0	-0.0874	
KURT	0.0	0.0	0.2924	0.0	0.0	0.1707	0.0	0.0	0.0	0.0	0.0	-0.0390	
MEAN	35.0000	43.0368	42.8449	35.0000	42.6179	42.6512	35.0000	42.3750	35.0000	42.3750	35.0000	42.4181	
VAR	19.2273	13.7378	15.7185	10.3343	7.5974	8.4607	5.6525	4.2083	5.6525	4.2083	5.6525	4.6399	
SKW	0.0	0.0	0.0081	0.0	0.0	0.1301	0.0	0.0	0.0	0.0	0.0	-0.0593	
KURT	0.0	0.0	1.5126	0.0	0.0	0.1241	0.0	0.0	0.0	0.0	0.0	-0.0013	
MEAN	35.0000	42.7631	42.1953	35.0000	43.9850	43.9987	35.0000	46.1013	35.0000	46.1013	35.0000	46.2043	
VAR	19.2273	13.6437	15.6959	10.3343	7.3820	8.5186	5.6525	3.1415	5.6525	3.1415	5.6525	6.7214	
SKW	0.0	0.0	-0.1241	0.0	0.0	-0.0648	0.0	0.0	0.0	0.0	0.0	0.0453	
KURT	0.0	0.0	0.3189	0.0	0.0	0.3419	0.0	0.0	0.0	0.0	0.0	0.0155	

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TABLE 1. SUMMARY STATISTICS FOR THE DISTRIBUTION OF PARAMETERS FOR BIVARIATE NORMAL  
 DISTRIBUTIONS, VARIOUS AND FIXED VALUES OF  $\rho$ , LARGE SAMPLE SIZES, FOR AMOUNTS OF REPLICATES, AND A VALUE  
 OF  $\rho$  FOR ESTIMATE BASED ON A CORRELATION OF 0.60

# SAMPLE SIZE

	N=5				N=13				N=37			
	EXPECT T	EXPECT O	OBSERVED	EXPECT T	EXPECT O	OBSERVED	EXPECT T	EXPECT O	OBSERVED	EXPECT T	EXPECT O	OBSERVED
MEAN	20.0000	20.0034	19.8697	20.0000	19.8367	19.9969	20.0000	19.9733	19.9448	20.0000	19.9733	19.9448
VAR	16.1245	17.0025	17.0883	8.6667	8.3971	8.3739	4.7403	4.7967	4.7978	4.7403	4.7967	4.7978
SKW	0.0	0.0	-0.2179	0.0	0.0	-0.0022	0.0	0.0	0.0164	0.0	0.0	0.0164
KURT	0.0	0.0	1.5062	0.0	0.0	0.4522	0.0	0.0	-0.0302	0.0	0.0	-0.0302
ERROR= 0												
MEAN	20.0000	19.6179	19.8130	20.0000	20.4023	20.5175	20.0000	20.1757	20.1192	20.0000	20.1757	20.1192
VAR	16.1245	15.4259	14.0403	8.6667	9.5614	9.2235	4.7403	4.9029	4.8374	4.7403	4.9029	4.8374
SKW	0.0	0.0	-0.0323	0.0	0.0	-0.0766	0.0	0.0	0.0066	0.0	0.0	0.0066
KURT	0.0	0.0	-0.1320	0.0	0.0	0.0144	0.0	0.0	0.0599	0.0	0.0	0.0599
ERROR= 33												
MEAN	20.0000	23.4199	23.4374	20.0000	22.6033	22.5590	20.0000	22.8499	22.9106	20.0000	22.8499	22.9106
VAR	16.1245	16.2421	15.5696	8.6667	8.6641	8.7138	4.7403	4.6652	4.7266	4.7403	4.6652	4.7266
SKW	0.0	0.0	0.1033	0.0	0.0	-0.1289	0.0	0.0	-0.0233	0.0	0.0	-0.0233
KURT	0.0	0.0	1.1206	0.0	0.0	0.3638	0.0	0.0	-0.1247	0.0	0.0	-0.1247
ERROR= 67												
MEAN	20.0000	22.2573	22.6871	20.0000	24.1920	24.6259	20.0000	27.2258	27.4992	20.0000	27.2258	27.4992
VAR	16.1245	14.8241	13.9714	8.6667	9.1010	9.0914	4.7403	4.5207	5.0165	4.7403	4.5207	5.0165
SKW	0.0	0.0	-0.1580	0.0	0.0	-0.1796	0.0	0.0	-0.0679	0.0	0.0	-0.0679
KURT	0.0	0.0	0.0676	0.0	0.0	-0.1464	0.0	0.0	0.0344	0.0	0.0	0.0344
ERROR= 100												
MEAN	20.0000	23.9234	29.1621	20.0000	29.7257	29.8039	20.0000	29.4946	29.4073	20.0000	29.4946	29.4073
VAR	16.1245	15.0772	15.5470	8.6667	8.0039	8.2217	4.7403	4.3064	4.4222	4.7403	4.3064	4.4222
SKW	0.0	0.0	0.5284	0.0	0.0	0.0873	0.0	0.0	-0.1130	0.0	0.0	-0.1130
KURT	0.0	0.0	2.4131	0.0	0.0	0.0001	0.0	0.0	0.0387	0.0	0.0	0.0387
ERROR= 100												
MEAN	20.0000	27.3720	29.0925	20.0000	32.2036	32.4539	20.0000	36.8586	37.1055	20.0000	36.8586	37.1055
VAR	16.1245	13.9472	15.0097	8.6667	8.0251	8.3503	4.7403	3.7344	5.7096	4.7403	3.7344	5.7096
SKW	0.0	0.0	-0.3627	0.0	0.0	-0.2390	0.0	0.0	0.0231	0.0	0.0	0.0231
KURT	0.0	0.0	1.1697	0.0	0.0	0.1610	0.0	0.0	-0.0919	0.0	0.0	-0.0919
ERROR= 100												
MEAN	20.0000	34.4240	34.6446	20.0000	35.0654	35.2079	20.0000	34.9207	34.9515	20.0000	34.9207	34.9515
VAR	16.1245	13.4755	14.4894	8.6667	7.1444	7.7479	4.7403	3.8915	4.3054	4.7403	3.8915	4.3054
SKW	0.0	0.0	0.1528	0.0	0.0	0.1300	0.0	0.0	-0.0096	0.0	0.0	-0.0096
KURT	0.0	0.0	0.8692	0.0	0.0	0.1110	0.0	0.0	0.3072	0.0	0.0	0.3072
ERROR= 100												
MEAN	20.0000	33.3851	34.8646	20.0000	38.7479	38.9360	20.0000	42.4165	42.8144	20.0000	42.4165	42.8144
VAR	16.1245	12.4443	14.2316	8.6667	6.7933	7.8200	4.7403	3.0214	6.8465	4.7403	3.0214	6.8465
SKW	0.0	0.0	-0.0129	0.0	0.0	-0.1040	0.0	0.0	0.0521	0.0	0.0	0.0521
KURT	0.0	0.0	0.3030	0.0	0.0	0.5152	0.0	0.0	0.0133	0.0	0.0	0.0133



		N=5				N=13				N=39			
		EXPECT	T	OSERVED	EXPECT	T	OSERVED	EXPECT	T	OSERVED	EXPECT	T	OSERVED
MEAN	5.0000	4.9176	5.3107	5.0000	4.9544	5.0002	5.0000	5.0347	5.0280				
VAR	8.7856	9.8212	9.1562	4.7221	4.3711	4.9714	2.5828	2.6025	2.6401				
SKW	0.0	0.0	0.1679	0.0	0.0	-0.0733	0.0	0.0	0.0276				
KURT	0.0	0.0	2.0527	0.0	0.0	0.0942	0.0	0.0	0.1256				
MEAN	5.0000	4.5147	4.5419	5.0000	5.0133	4.9469	5.0000	5.0046	4.9469				
VAR	8.7856	7.5482	7.6125	4.7221	5.1067	4.9437	2.5828	2.6495	2.6546				
SKW	0.0	0.0	-0.1534	0.0	0.0	-0.0635	0.0	0.0	0.0771				
KURT	0.0	0.0	1.0711	0.0	0.0	-0.1140	0.0	0.0	-0.1700				
MEAN	5.0000	5.5415	5.2584	5.0000	9.4868	9.5063	5.0000	3.3746	9.4206				
VAR	8.7856	10.1382	9.1259	4.7221	5.5547	5.4550	2.5828	2.9428	3.0025				
SKW	0.0	0.0	0.2674	0.0	0.0	-0.0421	0.0	0.0	-0.0660				
KURT	0.0	0.0	1.7536	0.0	0.0	0.1977	0.0	0.0	0.2533				
MEAN	5.0000	8.3973	9.6209	5.0000	10.8559	11.4514	5.0000	15.5527	16.0176				
VAR	8.7856	8.9619	9.2088	4.7221	5.8381	6.0095	2.5828	3.2959	4.6763				
SKW	0.0	0.0	-0.2943	0.0	0.0	-0.3973	0.0	0.0	-0.0690				
KURT	0.0	0.0	0.1566	0.0	0.0	0.4048	0.0	0.0	-0.0056				
MEAN	5.0000	13.7141	13.2959	5.0000	19.1323	19.1801	5.0000	19.0939	19.1251				
VAR	8.7856	12.0436	11.7561	4.7221	6.2005	6.3003	2.5828	3.3284	3.3276				
SKW	0.0	0.0	0.1020	0.0	0.0	0.1002	0.0	0.0	-0.0483				
KURT	0.0	0.0	1.2393	0.0	0.0	0.1707	0.0	0.0	-0.0490				
MEAN	5.0000	16.1528	13.4635	5.0000	23.5831	24.2166	5.0000	30.6148	31.2150				
VAR	8.7856	10.3412	11.3235	4.7221	6.3278	6.4923	2.5828	3.2272	6.0181				
SKW	0.0	0.0	-0.4441	0.0	0.0	-0.1902	0.0	0.0	0.0409				
KURT	0.0	0.0	0.9772	0.0	0.0	0.0314	0.0	0.0	0.0621				
MEAN	5.0000	27.4674	27.2419	5.0000	27.4984	27.5922	5.0000	27.5321	27.6139				
VAR	8.7856	11.1436	12.4505	4.7221	6.0926	6.5412	2.5828	3.3229	3.7015				
SKW	0.0	0.0	0.1070	0.0	0.0	-0.1144	0.0	0.0	-0.0503				
KURT	0.0	0.0	0.6301	0.0	0.0	-0.0856	0.0	0.0	0.1773				
MEAN	5.0000	25.2910	27.3282	5.0000	33.0360	33.4434	5.0000	33.4703	33.6870				
VAR													

		N=5				N=13				N=39			
		EXPECT	T	OBSERVED	EXPECT	T	OBSERVED	EXPECT	T	OBSERVED	EXPECT	T	RESERVED
MEAN	50.0000	49.9010	50.2251	50.0130	50.0000	50.0326	50.0130	50.0000	50.1996	50.1497			
VAR	20.1556	19.3035	19.2632	11.2463	10.8333	10.7002	11.2463	5.9254	5.9160	6.0236			
SKW	0.0	0.0	-0.0166	-0.1305	0.0	0.0	-0.1305	0.0	0.0	0.0436			
KURT	0.0	0.0	0.0017	0.0321	0.0	0.0	0.0321	0.0	0.0	0.1327			
ERROR= 0													
MEAN	50.0000	51.2472	51.1547	49.7130	50.0000	49.7994	49.7130	50.0000	50.0766	50.0943			
VAR	20.1556	13.9731	17.6760	9.7614	10.8333	10.2307	9.7614	5.9254	5.8262	5.5358			
SKW	0.0	0.0	0.0997	-0.0689	0.0	0.0	-0.0689	0.0	0.0	-0.1605			
KURT	0.0	0.0	0.0282	-0.2389	0.0	0.0	-0.2389	0.0	0.0	-0.0191			
ERROR= 33													
MEAN	50.0000	49.3041	49.3301	50.1534	50.0000	50.1457	50.1534	50.0000	50.0068	49.9344			
VAR	20.1556	13.8023	19.6545	10.3976	10.8333	10.2352	10.3976	5.9254	5.6172	5.8977			
SKW	0.0	0.0	0.0072	0.0003	0.0	0.0	0.0003	0.0	0.0	-0.0018			
KURT	0.0	0.0	1.1515	0.5205	0.0	0.0	0.5205	0.0	0.0	-0.0244			
ERROR= 33													
MEAN	50.0000	50.2344	50.2554	50.1745	50.0000	50.1304	50.1745	50.0000	49.9423	49.8893			
VAR	20.1556	13.1303	17.3502	9.2545	10.8333	9.8135	9.2545	5.9254	5.3184	5.3959			
SKW	0.0	0.0	0.0647	0.0484	0.0	0.0	0.0484	0.0	0.0	-0.0189			
KURT	0.0	0.0	0.3522	0.0459	0.0	0.0	0.0459	0.0	0.0	-0.1634			
ERROR= 67													
MEAN	50.0000	50.2367	50.2612	49.9432	50.0000	49.9795	49.9432	50.0000	49.9242	49.9749			
VAR	20.1556	16.7346	17.6345	9.5746	10.8333	9.0608	9.5746	5.9254	4.9582	5.3141			
SKW	0.0	0.0	0.0043	-0.0164	0.0	0.0	-0.0164	0.0	0.0	-0.0584			
KURT	0.0	0.0	0.6721	0.4370	0.0	0.0	0.4370	0.0	0.0	-0.1306			
ERROR= 67													
MEAN	50.0000	49.3530	49.5436	49.8322	50.0000	49.9726	49.8322	50.0000	49.8680	49.8351			
VAR	20.1556	16.3171	16.8972	8.6681	10.8333	8.8172	8.6681	5.9254	4.3373	4.7940			
SKW	0.0	0.0	-0.1547	0.0394	0.0	0.0	0.0394	0.0	0.0	0.0787			
KURT	0.0	0.0	0.7341	0.2460	0.0	0.0	0.2460	0.0	0.0	0.1380			
ERROR= 100													
MEAN	50.0000	49.2134	49.6186	50.2885	50.0000	50.4241	50.2885	50.0000	49.8623	49.8753			
VAR	20.1556	14.4075	15.4510	8.4280	10.8333	7.7724	8.4280	5.9254	4.2562	4.5171			
SKW	0.												

**1=33**

100

CC1=CC=C(C=C1)C

TABLE 17. SORT CHARACTERISTICS OF THE SAMPLING DISTRIBUTION OF INTERVALS FOR X VALUES REGULARLY  
 SPACED ABOUT THE Y VALUES. THEORETICALLY DISTRIBUTED FOR EVERY VALUE OF X, RANDOM AND FIXED VALUES  
 OF X, EQUAL SAMPLE SIZES, FOUR MOMENTS OF FPR, AND A VALUE OF FPR OF ESTIMATE BASED ON A  
 CORRELATION OF 0.00

SAMPLE SIZE

	N=5				N=13				N=39			
	MEAN	VAR	SKEN	KURT	MEAN	VAR	SKEN	KURT	MEAN	VAR	SKEN	KURT
FPR=0	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
FPR=33	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
FPR=67	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
FPR=100	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000
	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000	5.0000	5.0000	0.0000	0.0000

TABLE 18. SUMMARY STATISTICS OF THE CRITICAL STATISTICS FOR THE COMPARISON OF SLOPES AND INTERCEPTS  
RELATIVE TO SEVEN POPULATIONS, RANDOM AND FIXED VALUES OF  $\alpha$ , THREE SAMPLE SIZES, FOUR AMOUNTS OF  
CORRELATION, AND A VARIETY OF ERRORS IN ESTIMATE BASED ON A CORRELATION OF 0.9

## SAMPLE SIZE

N=5				N=13				N=39			
		EXPECTED OBSERVED		EXPECTED OBSERVED		EXPECTED OBSERVED		EXPECTED OBSERVED		EXPECTED OBSERVED	
N	SLOPE-MEAN F	1.5000	1.4260	1.1000	1.1193	1.0278	1.0390	1.0278	1.0390	1.0278	1.0390
	SLOPE-I ERROR	50/ 10	47/ 9	50/ 10	52/ 15	50/ 10	53/ 14	50/ 10	53/ 14	50/ 10	53/ 14
	INTERCEPT-MEAN F	1.4000	1.4975	1.0952	1.1147	1.0274	1.0297	1.0274	1.0297	1.0274	1.0297
	INTERCEPT-I ERROR	50/ 10	46/ 13	50/ 10	50/ 9	50/ 10	46/ 8	50/ 10	46/ 8	50/ 10	46/ 8
FN	SLOPE-MEAN F	1.5000	1.4473	1.1000	0.9953	1.0278	1.1174	1.0278	1.1174	1.0278	1.1174
	SLOPE-I ERROR	50/ 10	50/ 9	50/ 10	42/ 9	50/ 10	50/ 14	50/ 10	50/ 14	50/ 10	50/ 14
	INTERCEPT-MEAN F	1.4000	1.4863	1.0952	1.0386	1.0274	1.0917	1.0274	1.0917	1.0274	1.0917
	INTERCEPT-I ERROR	50/ 10	42/ 8	50/ 10	40/ 7	50/ 10	52/ 17	50/ 10	52/ 17	50/ 10	52/ 17
N	SLOPE-MEAN F	1.5000	1.3070	1.1000	1.1073	1.0278	1.0035	1.0278	1.0035	1.0278	1.0035
	SLOPE-I ERROR	50/ 10	44/ 5	50/ 10	63/ 9	50/ 10	39/ 8	50/ 10	39/ 8	50/ 10	39/ 8
	INTERCEPT-MEAN F	1.4000	1.3417	1.0952	1.1579	1.0274	0.9157	1.0274	0.9157	1.0274	0.9157
	INTERCEPT-I ERROR	50/ 10	44/ 7	50/ 10	56/ 8	50/ 10	48/ 14	50/ 10	48/ 14	50/ 10	48/ 14
FN	SLOPE-MEAN F	1.5000	1.3483	1.1000	1.0967	1.0278	1.0321	1.0278	1.0321	1.0278	1.0321
	SLOPE-I ERROR	50/ 10	43/ 8	50/ 10	54/ 13	50/ 10	47/ 9	50/ 10	47/ 9	50/ 10	47/ 9
	INTERCEPT-MEAN F	1.4000	1.4646	1.0952	1.1295	1.0274	1.0122	1.0274	1.0122	1.0274	1.0122
	INTERCEPT-I ERROR	50/ 10	44/ 10	50/ 10	43/ 6	50/ 10	49/ 10	50/ 10	49/ 10	50/ 10	49/ 10
N	SLOPE-MEAN F	1.5000	1.3557	1.1000	1.1030	1.0278	0.9851	1.0278	0.9851	1.0278	0.9851
	SLOPE-I ERROR	50/ 10	52/ 11	50/ 10	56/ 9	50/ 10	52/ 10	50/ 10	52/ 10	50/ 10	52/ 10
	INTERCEPT-MEAN F	1.4000	1.5364	1.0952	1.0346	1.0274	1.0352	1.0274	1.0352	1.0274	1.0352
	INTERCEPT-I ERROR	50/ 10	48/ 13	50/ 10	49/ 10	50/ 10	43/ 8	50/ 10	43/ 8	50/ 10	43/ 8
FN	SLOPE-MEAN F	1.5000	1.3282	1.1000	1.1571	1.0278	1.0200	1.0278	1.0200	1.0278	1.0200
	SLOPE-I ERROR	50/ 10	40/ 9	50/ 10	43/ 15	50/ 10	39/ 7	50/ 10	39/ 7	50/ 10	39/ 7
	INTERCEPT-MEAN F	1.4000	1.4764	1.0952	1.1159	1.0274	0.9615	1.0274	0.9615	1.0274	0.9615
	INTERCEPT-I ERROR	50/ 10	46/ 12	50/ 10	61/ 12	50/ 10	45/ 6	50/ 10	45/ 6	50/ 10	45/ 6
N	SLOPE-MEAN F	1.5000	1.5032	1.1000	1.0675	1.0278	1.0385	1.0278	1.0385	1.0278	1.0385
	SLOPE-I ERROR	50/ 10	51/ 13	50/ 10	56/ 13	50/ 10	48/ 13	50/ 10	48/ 13	50/ 10	48/ 13
	INTERCEPT-MEAN F	1.4000	1.4474	1.0952	1.1226	1.0274	1.0067	1.0274	1.0067	1.0274	1.0067
	INTERCEPT-I ERROR	50/ 10	53/ 12	50/ 10	35/ 11	50/ 10	52/ 10	50/ 10	52/ 10	50/ 10	52/ 10
FN	SLOPE-MEAN F	1.5000	1.3637	1.1000	1.1303	1.0278	0.9214	1.0278	0.9214	1.0278	0.9214
	SLOPE-I ERROR	50/ 10	53/ 12	50/ 10	50/ 7	50/ 10	61/ 16	50/ 10	61/ 16	50/ 10	61/ 16
	INTERCEPT-MEAN F	1.4000	1.5107	1.0952	1.0957	1.0274	1.0840	1.0274	1.0840	1.0274	1.0840
	INTERCEPT-I ERROR	50/ 10	43/ 12	50/ 10	46/ 12	50/ 10	34/ 4	50/ 10	34/ 4	50/ 10	34/ 4

ERROR= 33

128

ERROR= 67

ERROR=100

TABLE 10. SOME ESTIMATIONS OF THE CRITICAL STATISTICS FOR THE COMPARISON OF SLOPES AND INTERCEPTS  
IN REGRESSIONS OF Y ON X, FOR DIFFERENT SAMPLE SIZES, FOUR AMOUNTS OF  
VARIATION IN THE POPULATION, AND TWO AND FIXED VALUES OF X. THREE SAMPLE SIZES, FOUR AMOUNTS OF  
VARIATION IN THE POPULATION, AND ESTIMATES BASED ON A CORRELATION OF 0.30

## SAMPLE SIZE

	N=5		N=13		N=39	
	EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED
SLOPE=MEAN F	1.5000	1.4721	1.1000	1.1209	1.0278	1.0052
	50/ 10	51/ 9	50/ 10	47/ 12	50/ 10	58/ 12
	INTERCEPT=MEAN F	1.4000	1.0952	1.0392	1.0274	1.0640
	50/ 10	55/ 10	50/ 10	51/ 12	50/ 10	52/ 8
ERROR= 0						
SLOPE=MEAN F	1.5000	1.4380	1.1000	1.0765	1.0273	1.0415
	50/ 10	47/ 9	50/ 10	43/ 12	50/ 10	47/ 10
	INTERCEPT=MEAN F	1.4000	1.0952	1.1166	1.0274	1.0052
	50/ 10	51/ 14	50/ 10	48/ 7	50/ 10	47/ 9
SLOPE=MEAN F	1.5000	1.5160	1.1000	1.1299	1.0278	0.9792
	50/ 10	56/ 10	50/ 10	53/ 7	50/ 10	45/ 9
	INTERCEPT=MEAN F	1.4000	1.0952	1.1355	1.0274	1.0215
	50/ 10	45/ 6	50/ 10	52/ 9	50/ 10	47/ 8
ERROR= .33						
SLOPE=MEAN F	1.5000	1.4046	1.1000	1.0790	1.0278	1.0375
	50/ 10	43/ 8	50/ 10	43/ 5	50/ 10	50/ 9
	INTERCEPT=MEAN F	1.4000	1.0952	1.0349	1.0274	1.0045
	50/ 10	43/ 12	50/ 10	49/ 10	50/ 10	55/ 13
SLOPE=MEAN F	1.5000	1.3148	1.1000	1.0455	1.0278	0.9953
	50/ 10	41/ 4	50/ 10	48/ 13	50/ 10	54/ 14
	INTERCEPT=MEAN F	1.4000	1.0952	1.0752	1.0274	1.0016
	50/ 10	48/ 8	50/ 10	40/ 3	50/ 10	51/ 8
ERROR= .67						
SLOPE=MEAN F	1.5000	1.4333	1.1000	1.1104	1.0278	1.0250
	50/ 10	44/ 9	50/ 10	55/ 8	50/ 10	37/ 5
	INTERCEPT=MEAN F	1.4000	1.0952	1.0393	1.0274	1.0191
	50/ 10	55/ 10	50/ 10	48/ 10	50/ 10	49/ 13
SLOPE=MEAN F	1.5000	1.5678	1.1000	1.0323	1.0273	1.0284
	50/ 10	52/ 12	50/ 10	63/ 14	50/ 10	51/ 10
	INTERCEPT=MEAN F	1.4000	1.0952	1.1640	1.0274	1.0280
	50/ 10	64/ 13	50/ 10	43/ 8	50/ 10	44/ 11
ERROR=1.00						
SLOPE=MEAN F	1.5000	1.4202	1.1000	1.0345	1.0278	0.9811
	50/ 10	36/ 5	50/ 10	43/ 10	50/ 10	59/ 11
	INTERCEPT=MEAN F	1.4000	1.0952	1.0046	1.0274	1.1070
	50/ 10	52/ 15	50/ 10	47/ 5	50/ 10	40/ 8

$$\frac{5}{2}$$

CHICAGO

$$u_1, u_2, \dots, u_n = 33$$

19 = 67

$$C(0) = 0.0000$$
[illegible]



U.S. = 100

-31-

2. SOME COMPARISONS OF THE CRITICAL STATISTICS FOR THE COMPARISON OF SLOPES AND INTERCEPTS  
OF TWO LINEARLY DISTRIBUTED POPULATIONS WITH DIFFERENT VALUES NORMALLY DISTRIBUTED FOR EVERY VALUE OF  
SLOPE AND INTERCEPT, FOR DIFFERENT SAMPLE SIZES, FOR AMOUNTS OF ERROR, AND A VALUE OF ERROR OF  
ESTIMATE BASED ON A CORRELATION OF 0.0

## SAMPLE SIZE

	N=5		N=13		N=39	
	EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED
ERROR= 0	SLOPE-MEAN F	1.5000	1.5544	1.5100	1.0273	1.0436
	SLOPE-I ERROR	50/ 10	41/ 9	50/ 10	50/ 10	47/ 7
	INTERCEPT-MEAN F	1.4000	1.5257	1.0952	1.0274	1.0275
	INTERCEPT-I ERROR	50/ 10	60/ 13	50/ 10	50/ 10	55/ 15
	SLOPE-MEAN F	1.5000	1.5016	1.1000	1.0278	0.9995
	SLOPE-I ERROR	50/ 10	43/ 9	50/ 10	50/ 10	50/ 12
	INTERCEPT-MEAN F	1.4000	1.4510	1.0952	1.0274	1.0317
	INTERCEPT-I ERROR	50/ 10	44/ 12	50/ 10	50/ 10	44/ 9
ERROR= 33	SLOPE-MEAN F	1.5000	1.3972	1.1000	1.0278	0.9706
	SLOPE-I ERROR	50/ 10	53/ 12	50/ 10	50/ 10	50/ 13
	INTERCEPT-MEAN F	1.4000	1.6090	1.0952	1.0274	1.0923
	INTERCEPT-I ERROR	50/ 10	50/ 12	50/ 10	50/ 10	43/ 7
	SLOPE-MEAN F	1.5000	1.5133	1.1000	1.0278	1.0618
	SLOPE-I ERROR	50/ 10	51/ 9	50/ 10	50/ 10	38/ 4
	INTERCEPT-MEAN F	1.4000	1.5776	1.0952	1.0274	0.9393
	INTERCEPT-I ERROR	50/ 10	52/ 12	50/ 10	50/ 10	51/ 13
ERROR= 67	SLOPE-MEAN F	1.5000	1.3230	1.1000	1.0278	1.0258
	SLOPE-I ERROR	50/ 10	54/ 9	50/ 10	50/ 10	50/ 9
	INTERCEPT-MEAN F	1.4000	1.4951	1.0952	1.0274	1.0280
	INTERCEPT-I ERROR	50/ 10	48/ 10	50/ 10	50/ 10	62/ 17
	SLOPE-MEAN F	1.5000	1.5317	1.1000	1.0278	1.0838
	SLOPE-I ERROR	50/ 10	48/ 12	50/ 10	50/ 10	53/ 9
	INTERCEPT-MEAN F	1.4000	1.4874	1.0952	1.0274	1.0153
	INTERCEPT-I ERROR	50/ 10	47/ 11	50/ 10	50/ 10	59/ 16
ERROR=100	SLOPE-MEAN F	1.5000	1.4188	1.1000	1.0278	0.9670
	SLOPE-I ERROR	50/ 10	33/ 5	50/ 10	50/ 10	41/ 11
	INTERCEPT-MEAN F	1.4000	1.3316	1.0952	1.0274	1.0072
	INTERCEPT-I ERROR	50/ 10	55/ 11	50/ 10	50/ 10	38/ 8
	SLOPE-MEAN F	1.5000	1.2587	1.1000	1.0278	1.0235
	INTERCEPT-MEAN F	1.4000	1.5164	1.0952	1.0274	1.0947

$y = 39$ 

EXPECTED OBSERVED

1.3273 1.9195

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811C.1 + 47C.1

50/12 47/12

27 / 114 61 / 100

1.0273 1.8571

1.0218	0.0311
57/ 17	32/ 1

397 19 327 1  
1.0274 0.8384

1.0214 0.8384  
57/13 25/4

5/16/77 61/166

1-0273 0-9997

1.0218 0.9991  
57/17 47/9

397 13 417 9  
1-0274 1-0367

1.0214 1.0367  
53/17 53/7

1 /cc CT /ic

1 1273 1 8837

1.0273 J.8837  
53717 6377

[illegible]

1.0274 1.0114

C1 / 53 C1 / C5

1 0273 0 0737

1.0273	0.9737
537.13	461.9

53/ 13 44/ 8

1.0274 1.0023

50/ 10 '42/ 14

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1 0 3 8 1

1.0278 ) .9184

50/ 10 45/ 11

1.0274 1.0234

50/19 38/7

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1.0273 0.9816

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1.0274 1.0023

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1.0274 1.0273

9 / 64, C1 / C5

THESE CHARACTERISTICS OF THE CRITICAL STATISTICS FOR THE COMPARISON OF SLOPES AND INTERCEPTS  
 OF TWO REGRESSION LINES OBTAINED WITH THE Y VALUES NORMALLY DISTRIBUTED FOR EVERY VALUE OF  
 A GIVEN X VALUE OF X, WHEN SAMPLE SIZES, FOR BOTH S, ARE 25, AND A VALUE OF T-SCORE OF  
 0.60

SAMPLE SIZE

	n=5		n=13		n=39	
	EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED
S	SLOPE-MEAN F	1.5000	1.3583	1.1000	1.0278	1.0230
	SLOPE-I ERROR	50/ 10	43/ 7	50/ 10	50/ 10	51/ 14
	INTERCEPT-MEAN F	1.4000	1.4335	1.0952	1.0274	1.0922
	INTERCEPT-I ERROR	50/ 10	51/ 8	50/ 10	50/ 10	46/ 9
ERROR= 0						
F	SLOPE-MEAN F	1.5000	1.3796	1.1000	1.0278	1.0921
	SLOPE-I ERROR	50/ 10	54/ 13	50/ 10	50/ 10	53/ 10
	INTERCEPT-MEAN F	1.4000	1.6164	1.0952	1.0274	0.9861
	INTERCEPT-I ERROR	50/ 10	50/ 14	50/ 10	50/ 10	58/ 9
D	SLOPE-MEAN F	1.5000	1.4517	1.1000	1.0278	1.1025
	SLOPE-I ERROR	50/ 10	38/ 10	50/ 10	50/ 10	34/ 8
	INTERCEPT-MEAN F	1.4000	1.3871	1.0952	1.0274	0.9915
	INTERCEPT-I ERROR	50/ 10	54/ 9	50/ 10	50/ 10	66/ 16
R	SLOPE-MEAN F	1.5000	1.4397	1.1000	1.0278	1.1036
	SLOPE-I ERROR	50/ 10	50/ 19	50/ 10	50/ 10	55/ 15
	INTERCEPT-MEAN F	1.4000	1.5771	1.0952	1.0274	1.1138
	INTERCEPT-I ERROR	50/ 10	61/ 7	50/ 10	50/ 10	65/ 12
ERROR= 33						
R	SLOPE-MEAN F	1.5000	1.4578	1.1000	1.0278	1.0631
	SLOPE-I ERROR	50/ 10	37/ 4	50/ 10	50/ 10	43/ 10
	INTERCEPT-MEAN F	1.4000	1.3776	1.0952	1.0274	1.0321
	INTERCEPT-I ERROR	50/ 10	55/ 16	50/ 10	50/ 10	54/ 14
R	SLOPE-MEAN F	1.5000	1.4127	1.1000	1.0278	1.0815
	SLOPE-I ERROR	50/ 10	43/ 9	50/ 10	50/ 10	46/ 7
	INTERCEPT-MEAN F	1.4000	1.3370	1.0952	1.0274	0.9903
	INTERCEPT-I ERROR	50/ 10	50/ 13	50/ 10	50/ 10	53/ 12
R	SLOPE-MEAN F	1.5000	1.2724	1.1000	1.0278	1.0906
	SLOPE-I ERROR	50/ 10	53/ 9	50/ 10	50/ 10	51/ 10
	INTERCEPT-MEAN F	1.4000	1.4764	1.0952	1.0274	0.9916
	INTERCEPT-I ERROR	50/ 10	41/ 6	50/ 10	50/ 10	56/ 12
R	SLOPE-MEAN F	1.5000	1.1325	1.1000	1.0278	0.9734
	SLOPE-I ERROR	50/ 10	39/ 5	50/ 10	50/ 10	58/ 10
	INTERCEPT-MEAN F	1.4000	1.2070	1.0952	1.0274	1.1373
	INTERCEPT-I ERROR	50/ 10	40/ 9	50/ 10	50/ 10	41/ 5

ERROR=100

# SAMPLE SIZE

		N=5		N=13		N=39	
		EXPECTED	OBSERVED	EXPECTED	OBSERVED	EXPECTED	OBSERVED
		MEAN F		MEAN F		MEAN F	
R	SLOPE-MEAN F	1.5000	1.3880	1.1000	1.1667	1.0278	1.0786
	SLOPE-I ERROR	50/ 10	54/ 9	50/ 10	42/ 6	50/ 10	55/ 13
	INTERCEPT-MEAN F	1.4000	1.4573	1.0952	1.0328	1.0274	1.1175
	INTERCEPT-I ERROR	50/ 10	56/ 10	50/ 10	55/ 13	50/ 10	51/ 12
R=0							
R	SLOPE-MEAN F	1.5000	1.4501	1.1000	1.1097	1.0278	1.0324
	SLOPE-I ERROR	50/ 10	52/ 7	50/ 10	56/ 14	50/ 10	47/ 14
	INTERCEPT-MEAN F	1.4000	1.4045	1.0952	1.1086	1.0274	1.0100
	INTERCEPT-I ERROR	50/ 10	48/ 12	50/ 10	50/ 8	50/ 10	56/ 11
R=0.33							
R	SLOPE-MEAN F	1.5000	1.3065	1.1000	1.1154	1.0278	1.0621
	SLOPE-I ERROR	50/ 10	56/ 10	50/ 10	43/ 8	50/ 10	46/ 7
	INTERCEPT-MEAN F	1.4000	1.5174	1.0952	1.0839	1.0274	0.9735
	INTERCEPT-I ERROR	50/ 10	45/ 5	50/ 10	47/ 9	50/ 10	54/ 7
R=0.67							
R	SLOPE-MEAN F	1.5000	1.4552	1.1000	1.1090	1.0278	1.5058
	SLOPE-I ERROR	50/ 10	43/ 17	50/ 10	37/ 10	50/ 10	54/ 13
	INTERCEPT-MEAN F	1.4000	1.4270	1.0952	1.0286	1.0274	1.3705
	INTERCEPT-I ERROR	50/ 10	53/ 13	50/ 10	47/ 10	50/ 10	63/ 15
R=0.87							
R	SLOPE-MEAN F	1.5000	1.3529	1.1000	1.1135	1.0278	0.9075
	SLOPE-I ERROR	50/ 10	46/ 5	50/ 10	32/ 2	50/ 10	41/ 7
	INTERCEPT-MEAN F	1.4000	1.3202	1.0952	0.8984	1.0274	0.8923
	INTERCEPT-I ERROR	50/ 10	47/ 9	50/ 10	61/ 11	50/ 10	39/ 8
R=1.00							
R	SLOPE-MEAN F	1.5000	1.1533	1.1000	0.3564	1.0278	1.2317
	SLOPE-I ERROR	50/ 10	25/ 3	50/ 10	25/ 1	50/ 10	54/ 13
	INTERCEPT-MEAN F	1.4000	1.0495	1.0952	0.8126	1.0274	1.1081
	INTERCEPT-I ERROR	50/ 10	38/ 3	50/ 10	27/ 3	50/ 10	70/ 16
R=1.00							
R	SLOPE-MEAN F	1.5000	1.3439	1.1000	1.0831	1.0278	0.9836
	SLOPE-I ERROR	50/ 10	47/ 6	50/ 10	21/ 3	50/ 10	21/ 3
	INTERCEPT-MEAN F	1.4000	1.2008	1.0952	0.8364	1.0274	0.7992
	INTERCEPT-I ERROR	50/ 10	44/ 10	50/ 10	46/ 11	50/ 10	41/ 8
R=1.00							
R	SLOPE-MEAN F	1.5000	0.6014	1.1000	0.7567	1.0278	0.9633
	SLOPE-I ERROR	50/ 10	19/ 3	50/ 10	17/ 2	50/ 10	80/ 21
	INTERCEPT-MEAN F	1.4000	0.9509	1.0952	0.7204	1.0274	1.2824
	INTERCEPT-I ERROR	50/ 10	19/ 6	50/ 10	22/ 2	50/ 10	49/ 11

variable: (1) the concomitant variable is random rather than fixed; (2) the concomitant variable is fixed but contains error; and (3) the concomitant variable is random and contains error. The first of these three, which is a partial replication of the Calkins and Jennings (1972) study, can be examined by observing the results of the random and fixed sampling procedure (RN, random normal, and RR, random rectangular, versus FN, fixed normal, and FR, fixed rectangular) for the various experimental conditions when the concomitant variable contains no error. The second aspect, the concomitant variable is fixed but contains error, can be examined by observing the results of the fixed sampling procedure (FN, fixed normal, and FR, fixed rectangular) for the various experimental conditions when the concomitant variable contains varying amounts of error. The third aspect, the concomitant variable is random and contains error, can be examined by observing the results of the random sampling procedure (RN, random normal, and RR, random rectangular) for the various experimental conditions when the concomitant variable contains varying amounts of error.

Examination of Tables 2-25 reveals that considerable variation occurs for the observed values (OBSERVED) obtained in the experiments. For the variance, skewness, and kurtosis of the distributions of intercents and slopes, the means of the distributions of the critical statistics, and the number of Type I errors for intercents and slopes, the observed values seem to vary about the expected values (EXPECT T and EXPECT O) with no particularly important discernable pattern which can be attributed to the factors varied in the experiments, with one exception. The exception is evident for the distributions of intercent and slope for samples of size five where some of the values of skewness and kurtosis are of sufficient magnitude that it seems unlikely that these values are distributed normally. Thus it appears that neither (1) amount of error of estimate, (2) shape of the distribution of the

concomitant variable, (3) sampling procedure for the concomitant variable, (4) sample size (with the exception noted above), or (5) amount of error in the concomitant variable are related to any sizable disruption of (1) the critical statistics for contrasts of slopes and intercepts, or (2) the variance or normality of the distributions of the values of intercept and slope.

However, examination of Tables 2-17 reveals considerable disparity between expected values for central tendency for the distributions of intercept and slope (EXPECT T - MEAN), which are based on knowledge of the specified population parameters, and the corresponding observed central tendency values (OBSERVED - MEAN). Since the magnitude of this disparity increases with increasing amounts of error and seems unaffected by change in the other factors which were varied, it would seem reasonable to conclude that the estimates of the central tendency values for distributions of intercept and slope are disturbed by error in the concomitant variable. This conclusion, however, overlooks a crucial lack of control in the procedure - the lack of control over the variance of the concomitant variable after the introduction of the error component. When an expected central tendency value (EXPECT O) which was computed from the observed characteristics of the bivariate and marginal distributions is compared to the observed central tendency values, the magnitude of the discrepancy is greatly reduced, and the observed values again appear to vary about the expected values with no particularly important discernable pattern which can be attributed to the factors varied in the experiments. Thus it appears that when the results are viewed in such a way that the lack of control of the variance of the concomitant variable is compensated for, neither (1) amount of error of estimate, (2) shape of the distribution of the concomitant variable, (3) sampling procedure for the concomitant variable, (4) sample size, or probably (5) amount of error in

the concomitant variable is related to any systematic disruption of the estimates of the intercept and slope values.

### CONCLUSIONS

In the mathematical treatment of linear models, certain of the independent variables are assumed to be fixed variables (Graybill, 1961). When this assumption is made, it can be shown that the computed estimates of the parameters occurring in the models have the desirable characteristics that they are "good" estimates and are normally distributed.

It is often convenient to overlook this assumption when linear models are utilized in a research situation, since the nature of a variable often does not allow the researcher to select cases with particular values of a variable without discarding large amounts of data. This empirical study was undertaken as an attempt to discover the effects produced for the computed statistics and for the decisions made on the basis of a critical ratio concerning differences in these statistics for a particular family of linear models when certain independent variables are not fixed variables.

The assumption that certain independent variables have fixed values can be violated in one or both of two ways: (1) the researcher can fail to preselect values of the variables which will be found in the data and utilized to estimate the statistics, and/or (2) measurement error can be introduced through the process of observing the values of the independent variables. Calkins and Jennings (1972) reported that when certain independent variables are not fixed but are observed without error, the consequences of the violation of the assumption are negligible. The present study focused on the effects produced when the concomitant variable is either fixed or random but measured with error.

The particular family of linear models investigated are a set presented by Ward and Jennings (1973, pp. 138-148), who apply them to problems generally



approached by the use of the technique of analysis of covariance. The estimable terms in these models represent the estimates of the parameters of intercepts and slopes of group regression lines.

Three cases of the violation of the assumption that the concomitant variable is a fixed variable were investigated in this study. These three cases were (1) the concomitant variable is fixed but contains error, (2) the concomitant variable is random (partial replication of Calkins and Jennings, 1972), and (3) the concomitant variable is random and contains error. Three types of consequences of these three departures from the assumptions were investigated: (1) the estimates of the slope and intercept parameters are not "good," (2) the distributions of the values of these estimates are non-normal, and (3) the number of erroneous decisions concerning differences in the estimates are greater than expected.

In general the results of this study indicate that little discrepancy was evident between the observed and expected number of Type I errors resulting from any of the three cases of violation of the assumption. In addition, for none of the three cases did the distributions of the values of intercept and slope appear to be distributed other than normally except for samples of size five. Although there were differences of considerable magnitude between some of the expected and observed estimates of the intercept and slope parameters which appeared to be associated with amount of error in the concomitant variable, these differences were probably due to lack of control over the variance of the concomitant variable, since when the expected values were calculated utilizing the actual variance of the concomitant variable the magnitude of these differences was considerably reduced. Thus it appears likely that for none of these three cases of violation of the assumption are the estimates of the population values of intercept and slope seriously affected.

Probably only the tentative conclusion that the violation of the assumption that the concomitant variable be fixed and contain no error presents little or no problem for researchers should be drawn from this investigation. Before firmer conclusions are justified, the investigation should be repeated with better control of the variance of the concomitant variable when it contains error.

#### REFERENCES

- Calkins, D. S. and Jennings, Earl. An empirical investigation of some effects of the violation of the assumption that the covariable in analysis of covariance is a mathematical variable. Paper presented at the American Educational Research Association Annual Convention, 1972.
- Fisher, Ronald A. Statistical Methods for Research Workers (4th ed.). Edinburgh and London: Oliver and Boyd, Ltd., 1932.
- Fisher, Ronald A. Statistical Methods for Research Workers. New York: Hafner Publishing Company, 1973.
- Craybill, F. A. An Introduction to Linear Statistical Models. Vol. 1. New York: McGraw-Hill Book Company, Inc., 1961.
- Ward, Joe H. and Jennings, Earl. Introduction to Linear Models. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1973.